MEASUREMENT OF LEVEL-OF-SATISFACTION OF DECISION MAKER IN INTELLIGENT FUZZY-MCDM THEORY: A GENERALIZED APPROACH

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Abstract: The earliest definitions of decision support systems (DSS) identify DSS as systems to support managerial decision makers in unstructured or semi-unstructured decision situations. They are also defined as a computer-based information systems used to support decision-making activities in situations where it is not possible or not desirable to have an automated system perform the entire decision process. This chapter aims to delineate measurement of level-of-satisfaction during decision making under an intelligent fuzzy environment. Before proceeding with the multi-criteria decision making model (MCDM), authors try to build a co-relation among DSS, decision theories, and fuzziness of information. The co-relation shows the necessity of incorporating decision makers' level-of-satisfaction in MCDM models. Later, the authors introduce an MCDM model incorporating different cost factor components and the said level-of-satisfaction parameter. In a later chapter, the authors elucidate an application as well as validation of the devised model. The strength of the proposed MCDM methodology lies in combining both cardinal and ordinal information to get eclectic results from a complex, multi-person and multi-period problem hierarchically.

Key words: Decision support system, level-of-satisfaction in MCDM

C. Kahraman (ed.), Fuzzy Multi-Criteria Decision Making.

1. INTRODUCTION

Nomenclature

- D Decision matrix
- A Pair-wise comparison matrix among criteria (m × n)
- m Number of criteria
- n Number of alternatives of the pair-wise comparison matrix
- η_{max} Principal eigen value of "A" matrix
- PV Priority vector
- I.I. Inconsistency index of "A" matrix
- R.I. Random inconsistency index of "A" matrix
- I.R. Inconsistency ratio of "A" matrix
- α Level of satisfaction of decision maker
- OFM Objective factor measure
- SFM Subjective factor measure
- OFC Objective factor cost
- SI Selection index
- γ Fuzzy parameter that measures the degree of vagueness; $\gamma = 0$ indicates crisp.

1.1 DSS and Their Components

Decision support systems (DSS) can be defined as computer-based information systems that aid a decision maker in making decisions for semi-structured problems. Numerous definitions to DSS exist. The earliest definitions of DSS (Gorry and Morton, 1977) identify DSS as systems to support managerial decision makers in unstructured or semi-unstructured decision situations. Ginzberg and Stohr (1981) propose DSS as "a computer-based information system used to support decision making activities in situations where it is not possible or not desirable to have an automated system performs the entire decision process." However, the most apt working definition is provided by Turban (1990). According to Turban (1990) "a DSS is an interactive, flexible, and adaptable computer based information system that utilizes decision rules, models, and model base coupled with a comprehensive database and the decision maker's own insights, leading to specific, implementable decisions in solving problems that would not be amenable to management science models per se. Thus, a DSS supports complex decision making and increases its effectiveness." Alter (2004) explores the assumption that stripping the word *system* from DSS, focusing on decision support, and using ideas related to the work system method might generate some interesting directions for research and practice. Some of these directions fit under the DSS umbrella, and some seem to be excluded because they are not directly related to a technical artifact called a DSS. Alter (2004) suggests that "decision support is the use of any plausible computerized or non-computerized means for improving sense making and/or decision making in a particular repetitive or non-repetitive business situation in a particular organization."

However, the main objectives of DSS can be stated as follows:

1. To provide assistance to decision makers in situations that are semi-structured,
2. To identify plans and potential actions to resolve problems,
3. To rank the solutions identified that can be implemented and provide a list of viable alternatives.

DSS attempts to bring together and focus several independent disciplines. These are as follows:

1. Operations research (OR),
2. Management science (MS),
3. Database technology,
4. Artificial intelligence (AI),
5. Systems engineering,
6. Decision analysis.

Artificial intelligence is a field of study that attempts to build software systems exhibiting near-human "intellectual" capabilities. Modern works on AI are focused on fuzzy logic, artificial neural networks (ANNs), and genetic algorithms (GAs). These works, when integrated with DSS, enhance the performance of making decisions. AI systems are used in creating intelligent models, analyzing models intelligently, interpreting results found from models intelligently, and choosing models appropriately for specific applications.

Decision analysis may be divided into two major areas. The first, descriptive analysis, is concerned with understanding how people actually make decisions. The second, normative analysis, attempts to prescribe how people should make decisions. Both are issues of concern to DSS. The central aim of decision analysis is improving decision making processes.

Decisions, in general, are classified into three major categories:

- Structured decisions,
- Unstructured decisions,
- Semi-structured decisions.

Structured decisions are those decisions where all steps of decision making are well structured. Computer code generation is comparatively easy for these types of decisions.

In unstructured decisions, none of the steps of decision making is structured. AI systems are being built up to solve the problems of unstructured decisions.

Semi-structured decisions comprise characteristics of structured and unstructured decisions.

The DSS framework contains two types of components, which may be used either individually or in tandem. The first component is a multi-objective programming (MOP) model, which employs mathematical programming to generate alternative mitigation plans. Typically, an MOP model must be formulated for the specific problem at hand, but once formulated, it can be solved on a computer using commercially available software. The second component is a multi-criteria decision making (MCDM) model, used for evaluating decision alternatives that have been generated either by the MOP model or by some other method. MCDM models are typically “shells” that can be applied to a wide range of problem types. A variety of MCDM methodologies exist, some of which are available in the form of commercial software. A manufacturing information system can also be used in conjunction with the DSS, for both managing data and for compiling decisions of alternative plans generated by the DSS.

1.2 Decision-Making Processes

Strategic, tactical, and operative decisions are made on the various aspects of business operations. The vision of an industrial enterprise must take into consideration the possible changes in its operational environment, strategies, and the leadership practices. Decision making is supported by analyses, models, and computer-aided tools. Technological advances have an impact on the business of industrial enterprises and on their uses of new innovations. Industrial innovations contribute to increased productivity and the diversification of production and products; they help to create better, more challenging jobs and to minimize risks.

Long-term decisions have an impact on process changes, functional procedures and maintenance and also on safety, performance, costs, human factors and organisations. Short-term decisions deal with daily actions and their risks. Decision-making is facilitated by an analysis that incorporates a classification of one’s own views, calculating numerical values, translating the results of analysis into concrete properties and a numerical evaluation of the properties. One method applied for this purpose is the Analytic Hierarchy Process (AHP) model (Saaty, 1990). This model, which has many features in common with the other MCDMs applied in the current research work, is suited for manufacturing decision making processes that aim at making the correct choices in both the short and the long term.

1.3 MCDM

According to Agrell (1995) MCDM offers the methodology for decision making analysis when dealing with multiple objectives. This may be the case when the success of the application depends on the properties of the system, the decision maker, and the problem. Problems with engineering design involve multiple criteria: the transformation of resources into artifacts, a desire to maximize performance, and the need to comply with specifications.

The MCDM methodology can be used to increase performance and to decrease manufacturing costs and delays of enterprises. The Multiple-Criteria Decision Support System (MC-DSS) uses the MCDM methodology and ensures mathematical efficiency. The system employs graphical presentations and can be integrated with other design tools. Modeling and analyzing complex systems always involve an array of computational and conceptual difficulties, whereas a traditional modeling approach is based primarily on simulation and concepts taken from control theory.

The strength of the MCDM lies in the systematic and quantitative framework it offers to support decision making. Comprehensive tuning or parametric design of a complex system requires elaboration on using the modeling facilities of system dynamics and on the interactive decision making support of the MCDM.

Most experienced decision makers do not rely on a theory to make their decisions because of cumbersome techniques involved in the process of making decisions. But analytic decision making is of tremendous value when the said analytic process involves simple procedures and is accessible to the lay user as well as it possesses meaningful scientific justification of the highest order (Saaty, 1994).

The benefits of descriptive analytical approaches for decision making are as follows (Saaty, 1994):

1. To permit decision makers to use information relating to decision making in a morphological way of thoroughly modeling the decision and to make explicit decision makers' tactical knowledge;
2. To permit decision makers to use judgments and observations in order to surmise relations and strengths of relations in the flow of interacting forces moving from the general to the particular and to make predictions of most likely outcomes;
3. To enable decision makers to incorporate and trade off attribute values;
4. To enable decision makers to include judgments that result from intuition, day-to-day experiences, as well as those that result from logic;
5. To allow decision makers to make gradual and more thorough revisions and to combine the conclusions of different people studying the same problem in different places.

1.4 Information *vis-à-vis* MCDM Theories

Information is a system of knowledge that has been transformed from raw "data" into some meaningful form. Data are the raw materials for information. Data are also expressions of "events." Information has value in current or prospective decision making at a specified time and place for taking appropriate "action" resulting in evaluation of "performance." In this context attention is drawn to Figure 1. The terms "data" and "information" are often used interchangeably, but there is a distinction in that. Data are processed to provide information, and the information is related to decision making (Davis, 1974). A schematic diagram illustrating relationship between data and information is shown in Figure 2. If there is no need for making decisions, information would be unnecessary.

Information is the currency of the new economy. Yet most real-world cases lack the means to effectively organize and distribute the information their employees need to make quick, smart business decisions. A structured, personalized, self-serve way to access information and collaborate across departmental and geographical boundaries provides the basic needs for making a good decision.

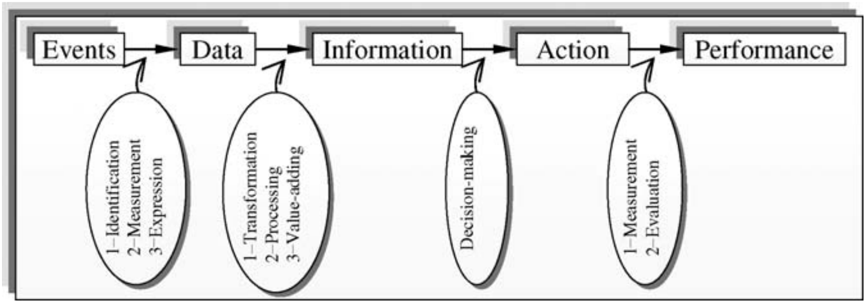

Figure 1. Generation and utilization of information

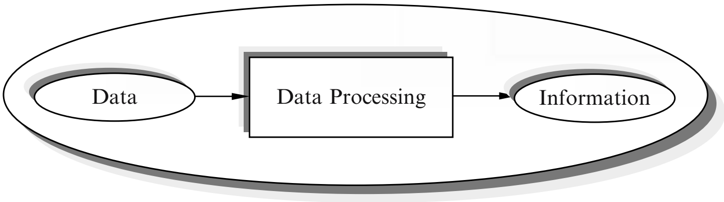

Figure 2. Converting raw data by an information system into useful information

1.5 Hidden Parameters in Information

1.5.1 Uncertainty in Information

Uncertainty permeates understanding of the real world. The purpose of information systems is to model the real world. Hence, information systems must be able to deal with uncertainty.

Many information systems include capabilities for dealing with some kinds of uncertainty. For example, database systems can represent missing values, information retrieval systems can match information to requests using a "weak" matching algorithm, and expert systems can represent rules that are known to be true only for "most" or "some" of the time. By and large, commercial information systems (e.g., database systems, information retrieval systems, or expert systems) have been slow to incorporate capabilities for dealing with uncertainty.

Uncertainty also has a long history of being associated with decision making research as Harris (1998) notes:

> Decision making is the process of sufficiently reducing uncertainty and doubt about alternatives to allow a reasonable choice to be

> made from among them. This definition stresses the information gathering function of decision making. It should be noted here that uncertainty is reduced rather than eliminated. Very few decisions are made with absolute certainty because complete knowledge about all the alternatives is seldom possible.

Researchers in various fields have also been concerned with the relationship between uncertainty and information seeking. In information science, the idea of uncertainty underlies all aspects of information seeking and searching. Kuhlthau (1993) has proposed uncertainty as a basic principle for information seeking, defining uncertainty as "a cognitive state which commonly causes affective symptoms of anxiety and lack of confidence." And, drawing on her research, she notes that, "Uncertainty and anxiety can be expected in the early stages of the information search process.… Uncertainty due to a lack of understanding, a gap in meaning, or a limited construct initiates the process of information seeking."

One of the biggest challenges for a manufacturing decision maker is the degree of uncertainty in the information that he or she has to process. In making some decisions, this is especially obvious when experts in the same area provide conflicting opinions on the attributes meant for making decisions. Disagreement among experts making decisions results in conflicting effects information. The decision maker is likely to place increased importance on the source of the information. This in itself is not surprising, but the battle of the credentials that follows perhaps is. There seems to be a danger that the may come to rely on the reputation of an expert, rather than on ensuring thorough scrutiny of the information that he or she has provided.

Actors in the decision making process may use uncertainty in the *effects*, and information as a means to promote their attributes. A proponent can try to downplay the *effects* of a development because they may not occur, whereas those in opposition may attempt to stall a project claiming that the disputed *effects* are likely to happen and are serious in nature. The decision maker is then left with the difficult task of navigating these disparities to come to a decision. In particular in the face of uncertainty, there seems to be a human tendency to make personal observations the deciding factor.

1.5.1.1 Sources of Uncertainty

Uncertainties are solely due to the unavailability of "perfect" information. Uncertainty might result from using unreliable information sources, for example, faulty reading instruments, or input forms that have been filled out incorrectly (intentionally or inadvertently). In other cases, uncertainty is a result of system errors, including input errors, transmission “noise,” delays in processing update transactions, imperfections of the system software, and corrupted data owing to failure or sabotage. At times, uncertainty is the unavoidable result of information gathering methods that require estimation or judgment.

In other cases, uncertainty is the result of restrictions imposed by the model. For example, if the database schema permits storing at most two occupations per employee, descriptions of occupation would exhibit uncertainty. Similarly, the sheer volume of information that is necessary to describe a real-world object might force the modeler to turn to approximation and sampling techniques.

1.5.1.2 Degree of Uncertainty

The relevant information that is available in the absence of certain information may take different forms, each exhibiting a different level of uncertainty. Uncertainty is highest when the mere existence of some real-world object is in doubt. The simplest solution is to ignore such objects altogether. This solution, however, is unacceptable if the model claims to be closed world (i.e., objects not modeled do not exist).

Uncertainty is reduced somewhat when each element is assigned a value in a prescribed range, to indicate the certainty that the modeled object exists. When the element is a fact, this value can be interpreted as the confidence that the fact holds; when it is a rule, this value can be interpreted as the strength of the rule (percent of cases where the rule applies).

Now it is assumed that “existence” is assured, but some or all of the information with which the model describes an object is unknown. Such information has also been referred to as incomplete, missing, or unavailable.

Uncertainty is reduced when the information that describes an object is known to come from a limited set of alternatives (possibly a range of values). This uncertainty is referred to as disjunctive information. Note that when the set of alternatives is simply the entire “universe,” this case reverts to the previous (less informative) case.

Uncertainty is reduced even more when each alternative is accompanied by a number describing the probability that it is indeed the true description (and the sum of these numbers for the entire set is 1). In this case, the uncertain information is probabilistic. Again, when the probabilities are unavailable, probabilistic information becomes disjunctive information.

Occasionally, the information available to describe an object is descriptive rather than quantitative. Such information is often referred to as fuzzy or vague information.

1.5.1.3 Vagueness in Information

Russell (1923) attributes vagueness to being mostly a problem of language. Of course, language is part of the problem, but it is not the main problem. There would still be vagueness even if we had a very precise, logically structured, language. The principal source of vagueness seems to be in making discreet statements about continuous phenomenon. According to Russell (1923), “Vagueness in a cognitive occurrence is a characteristic of its relation to that which is known, not a characteristic of the occurrence in itself.” Russell (1923) adds, “Vagueness, though it applies primarily to what is cognitive, is a conception applicable to every kind of representation.”

Surprisingly, Wells (1908) was among the first to suggest the concept of vagueness:

> Every species is vague, every term goes cloudy at its edges, and so in my way of thinking, relentless logic is only another name for stupidity for a sort of intellectual pigheadedness. If you push a philosophical or metaphysical enquiry through a series of valid syllogisms never committing any generally recognized fallacy you nevertheless leave behind you at each step a certain rubbing and marginal loss of objective truth and you get deflections that are difficult to trace, at each phase in the process. Every species waggles about in its definition, every tool is a little loose in its handle, every scale has its individual.

In real-world problems there is always a chance of getting introduced to the vagueness factor when information deals in combination with both cardinal and ordinal measures. It should always be remembered that reduction of vagueness is to be addressed in a situation where decision alternatives are well inter-related and have both cardinal and ordinal criteria for selection.

1.5.1.4 Sources of Vagueness

Linguistic expressions in classic decision making processes incorporate unquantifiable, imperfect, nonobtainable information and partially ignorant facts. Data combining both ordinal and cardinal preferences in real-world decision making problems are highly unreliable and both contain a certain degree of vagueness. Crisp data often contains some amount of vagueness and, therefore, need the attention of decision makers in order to achieve a lesser degree of vagueness inherent.

The purpose of decision making processes is best served when imprecision is communicated as precisely as possible but no more precisely than warranted.

2. PRIOR WORKS ON FUZZY-MCDM FOR SELECTING BEST CANDIDATE-ALTERNATIVE

The available literature on MCDM tackling fuzziness is as broad as it is diverse. Literature contains several proposals on how to incorporate the inherent uncertainty as well as the vagueness associated with the decision maker's knowledge into the model (Arbel, 1989; Arbel and Vargas, 1990; Banuelas and Antony, 2004; Saaty and Vargas, 1987). The analytic hierarchy process (AHP) (Saaty, 1980 and 1990) literature, in this regard, is also vast.

There has been a great deal of interest in the application of fuzzy sets to the representation of fuzziness and uncertainty in management decision models (Buckley, 1988; Chen and Hwang, 1982; Ghotb and Warren, 1995; Gogus and Boucher, 1997; Van Laarhoven and Pedrycz, 1983; Liang and Wang, 1994; Lai and Hwang, 1994; Zimmerman, 1976, 1987). Some approaches were made to handle the uncertainties of MCDM problems. Bellman and Zadeh (1970) have shown fuzzy set theory's applicability to the MCDM study. Yager and Basson (1975) and Bass and Kwakernaak (1977) have introduced maximin and simple additive weighing model using the membership function (MF) of the fuzzy set. Most of the recent literature is filled with mathematical proofs.

A decision maker needs an MCDM assessment technique in regard to its fuzziness that can be easily used in practice. An approach was taken earlier by Marcelloni and Aksit (2001). Their aim was to model inconsistencies through the application of fuzzy logic-based techniques. Boucher and Gogus (2002) examined certain characteristics of judgment elicitation instruments appropriate to fuzzy MCDM. In their work the fuzziness was measured using a gamma function.

By defining a decision maker's preference structure in fuzzy linear constraint (FLC) with soft inequality, one can operate the concerned fuzzy optimization model with a modified *S*-curve smooth MF to achieve the desired solution (Watada, 1997). One form of logistic MF to overcome difficulties in using a linear membership function in solving a fuzzy decision making problem was proposed by Watada (1997). However, it is expected that a new form of logistic membership function based on nonlinear properties can be derived, and its flexibility in fitting real-life problem parameters can be investigated. Such a formulation of a nonlinear logistic MF was presented in this work, and its flexibility in taking up the fuzziness of the parameter in a real-life problem was demonstrated.

Carlsson and Korhonen (1986) have illustrated, through an example, the usefulness of a formulated MF, viz., an exponential logistic function. Their illustrated example was adopted to test and compare a nonlinear MF (Lootsma, 1997). Such an attempt using the said validated nonlinear MF and comparing the results was made by Vasant et al. (2005). Comprehensive tests based on a real-life industrial problem have to be undertaken on the newly developed membership function in order to prove further its applicability in fuzzy decision making (Vasant, 2003; Vasant et al., 2002; 2005). To test the newly formulated MF in problems as stated above, a software platform is essential. In this work MATLAB has been chosen as the software platform using its M-file for greater flexibility.

In the past, studies on decision making problems were considered on the bipartite relationship of the decision maker and analyst (Tabucanon, 1996). This is with the assumption that the implementers are a group of robots that are programmed to follow instructions from the decision maker. This notion is now outdated. Now a tripartite relationship is to be considered, as shown on Figure 3, where the decision maker, the analyst, and the implementer will interact in finding a fuzzy satisfactory solution in any given fuzzy system. This is because the implementers are human beings, and they have to accept the solutions given by the decision maker to be implemented under a turbulent environment.

In case of tripartite fuzzy systems, the decision maker will communicate and describe the fuzzy problem with an analyst. Based on

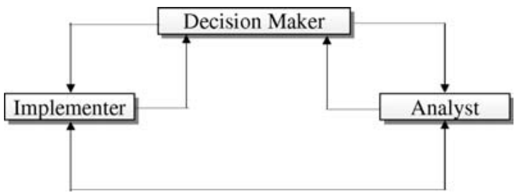

Figure 3. Tripartite relationship for MCDM problems

the data that are provided by the decision maker, the analyst will formulate MFs, solve the fuzzy problems, and provide the solution back to the decision-maker. After that, the decision maker will provide the fuzzy solution with a trade off to the implementer for implementation. An implementer has to interact with decision maker to obtain an efficient and highly productive fuzzy solution with a certain degree of satisfaction. This fuzzy system will eventually be called a high productive fuzzy system (Rommelfanger, 1996). A tripartite relationship, decision maker−analyst−implementer, is essential to solve any industrial problem.

The following criticisms of the existing literatures, in general, are made after a study of the existing vast literature on the use of various types of MFs in finding out fuzziness patterns of MCDM methodologies:

1. Data combining both ordinal and cardinal preferences contain non-obtainable information and partially ignorant facts. Both ordinal and cardinal preferences contain a certain degree of fuzziness and are highly unreliable, unquantifiable and imperfect.
2. Simplified fuzzy MFs, viz., trapezoidal and triangular and even gamma functions, are not able to bring out real-world fuzziness patterns in order to elucidate a degree of fuzziness inherent in the MCDM model.
3. Level-of-satisfaction of the decision makers should be judged through a simple procedure while making decisions through MCDM models.
4. An intelligent tripartite relationship among the decision maker, analyst and implementer is essential, in conjunction to a more flexible MF design, to solve any real-world MCDM problem.

Among many diversified objectives of the current work, one objective is to find out fuzziness patterns of the candidate-alternatives having disparate level-of-satisfaction in MCDM model. Relationships among the degree of fuzziness, level-of-satisfaction and the selection-indices of the MCDM model guide decision makers under a tripartite fuzzy environment in obtaining their choice tradeoff with a predetermined allowable imprecision.

Another objective of the current work is to provide a robust, quantified monitor of the level-of-satisfaction among decision makers and to calibrate these levels of satisfaction against decision makers' expectations. Yet another objective is to provide a practical tool for further assessing the impact of different options and available courses of action.

3. COMPONENTS OF THE MCDM MODEL

The proposed MCDM model considers a fuzziness pattern in disparate level-of-satisfaction of the decision maker. The model outlines a MF for evaluating degree of fuzziness hidden in the Eq. (1). AHP provides the decision maker's with a vector of priorities (PV) to estimate the expected utilities of each candidate-FMS.

A mathematical model was proposed by Bhattacharya et al. (2004, 2005) to combine cost factor components with the importance weightings found from AHP. The governing Eq. of the said model is:

OFM = Objective factor measure,
OFC = Objective factor cost,
SFM = Subjective factor measure,
SI = Selection index,
α = Objective factor decision weight,
n = Finite number of candidate-alternative.

$$\mathrm{SI}_i = \left[\left(\alpha \times \mathrm{SFM}_i\right) + \left(1-\alpha\right) \times \mathrm{OFM}_i\right] \tag{1}$$

where

$$\mathrm{OFM}_l = \frac{1}{\mathrm{OFC}_l \times \sum_{l=1}^{n} \mathrm{OFC}^l} \tag{2}$$

In the said model, AHP plays a crucial role. AHP is an MCDM method, and it refers to making decisions in the presence of multiple, usually conflicting, criteria. A criterion is a measure of effectiveness. It is the basis for evaluation. Criteria emerge as a form of attributes or objectives in the actual problem setting. In reality, multiple criteria usually conflict with each other. Each objective/attribute has a different unit of measurement. Solutions to the problems by AHP are either to design the best alternative or to select the best one among the previously specified finite alternatives.

For assigning the weights to each of the attributes as well as to the alternative processes for constructing the decision matrix and pair-wise comparison matrices, the phrase like "much more important" is used to extract the decision maker's preferences. Saaty (1990) gives an intensity scale of importance (refer to Table 1) and has broken down the importance ranks.

Table 1. The Nine-Point Scale of Pair-Wise Comparison

Intensity scale	Interpretation
1	Equally important
3	Moderately preferred
5	Essentially preferred
7	Very strongly preferred
9	Extremely preferred
2, 4, 6, 8	Intermediate importance between two adjacent judgments

In AHP the decision matrix is always a square matrix. Using the advantage of properties of eigenvalues and eigenvectors of a square matrix, the level of inconsistency of the judgmental values assigned to each elements of the matrix is checked.

In this chapter the proposed methodology is applied to calculate the priority weights for functional, design factors and other important attributes by eigenvector method for each pair-wise comparison matrix. Next, global priorities of various attributes rating are found by using AHP. These global priority values are used as SFM in Eq. (1). The pair-wise comparison matrices for five different factors are constructed on the basis of Saaty's nine-point scale (refer to Table 1). The objective factors, i.e., OFM, and OFC are calculated separately by using cost factor components.

In the mathematical modeling for finding the SFM_i values, decomposition of the total problem (factor-wise) into smaller sub-problems has been done. This is done so that each sub-problem can be analyzed and appropriately handled with practical perspectives in terms of data and information. The objective of decomposition of the total problem for finding out the SFM values is to enable a pair-wise comparison of all the elements on a given level with respect to the related elements in the level just above.

The proposed algorithm consists of a few steps of calculations. Prior to the calculation part, listing of the set of candidate-alternatives is carried out. Next, the cost components of the candidate-alternatives are quantified. Factors, on which the decision making is based, are identified as intrinsic and extrinsic. A graphical representation depicting the hierarchy of the problem in terms of overall objective, factors, and number of alternatives is to be developed. Next follows the assigning of the judgmental values to the factors as well as to the candidate-alternatives to construct the decision matrix and pair-wise comparison matrices, respectively.

A decision matrix is constructed by assigning weights to each factor based on the relative importance of its contribution according to a nine-point scale (refer to Table 1). Assigning the weights to each candidate-alternative for each factor follows the same logic as that of the decision matrix. This matrix is known as a pair-wise comparison matrix. The PV values are determined then for both the decision and the pair-wise comparison matrices. The $\eta_{\max}$ for each matrix may be found by multiplication of the sum of each column with the corresponding PV value and subsequent summation of these products.

There is a "check" in the judgmental values given to the decision and pair-wise comparison matrices for revising and improving the judgments. If I.R. is greater than 10%, the values assigned to each element of the decision and pair-wise comparison matrices are said to be inconsistent. For I.R. < 10%, the level of inconsistency is acceptable. Otherwise the level of inconsistency in the matrices is high and the decision maker is advised to revise the judgmental values of the matrices to produce more consistent matrices. It is expected that all the comparison matrices should be consistent. But the very root of the judgment in constructing these matrices is the human being. So, some degree of inconsistency of the judgments of these matrices is fixed at 10%. Calculation of I.R. involves I.I., R.I., and I.R. These matrices are evaluated from Eqs. (3), (4) and (5) respectively.

$$\text{I.I.} = \frac{(\eta_{\max} - n)}{(n - 1)} \tag{3}$$

$$\text{R.I.} = \frac{[1.98 \times (n - 2)]}{n} \tag{4}$$

$$\text{I.R.} = \frac{\text{I.I.}}{\text{R.I.}} \tag{5}$$

The OFM_i values are determined by Eq (6).

$$\text{OFM}_i = [\text{OFC}_i \times \sum_{i=1}^{n} \frac{1}{\text{OFC}_i}]^{-1} \tag{6}$$

The SFM_i values are the global priorities for each candidate-alternative. SFM_i may be found by multiplying each of the decision matrix PV values to each of the PV value of each candidate-alternative for each factor. Each product is then summed up for each alternative to get SFM_i.

For an easy demonstration of the proposed fuzzified MCDM model, efforts for additional fuzzification are confined assuming that differences in judgmental values are only 5%. Therefore, the upper bound and lower bound of SFM_i as well as SI_i indices are to be computed within a range of 5% of the original values. In order to avoid complexity in delineating the technique proposed hereinbefore, we have considered the 5% measurement. One can fuzzify the SFM_i values from the very beginning of the model by introducing a modified *S*-curve MF in AHP, and the corresponding fuzzification of SI_i indices can also be carried out using the holistic approach used in Eq. (1). The set of candidate-alternatives are then ranked according to the descending order of SI_i indices (refer to Eq. 7).

$$\tilde{LSI}_i\Big|_{\alpha=\alpha_{SFM_i}} = LSI_L + \left(\frac{LSI_U - LSI_L}{\gamma}\right)\ln\frac{1}{C}\left(\frac{A}{\alpha_{LSI_i}} - 1\right) \tag{7}$$

In this work, a monotonically nonincreasing logistic function has been used as a membership function:

$$f(x) = \frac{B}{1 + Ce^{\gamma x}} \tag{8}$$

where α is the level-of-satisfaction of the decision maker; *B* and *C* are scalar constants; and γ, $0 < \gamma < \infty$ is a fuzzy parameter that measures the degree of vagueness (fuzziness), wherein $\gamma = 0$ indicates crisp. Fuzziness becomes highest when $\gamma \to \infty$.

The generalized logistic membership function is defined as

$$f(x) = \begin{cases} 1 & x < x_L \\ \dfrac{B}{1 + Ce^{\gamma x}} & x_L < x < x_U \\ 0 & x > x_U \end{cases} \tag{9}$$

To fit into the MCDM model in order to sense its degree of fuzziness, the Eq. (9) is modified and redefined as follows:

$$\mu(x) = \begin{cases} 1 & x < x^a \\ 0.999 & x = x^a \\ \dfrac{B}{1 + Ce^{\gamma x}} & x^a < x < x^b \\ 0.001 & x = x^b \\ 0 & x > x^b \end{cases} \tag{10}$$

In Eq. (10) the membership function is redefined as $0.001 \le \mu(x) \le 0.999$. This range is selected because in real-world situations the workforce need not be always 100% of the requirement. At the same time the workforce will not be 0%. Therefore, there is a range between x_0 and x_1 with $0.001 \leq \mu(x) \leq 0.999$. This concept of range of $\mu(x)$ is used in this chapter.

Choice of the level-of-satisfaction of the decision maker, i.e., α, is an important issue. It is the outcome of the aggregate decision by the design engineer, production engineer, maintenance engineer, and capital investor of a manufacturing organization. However, the selection of a candidate-alternative may give different sets of results for different values of α for the same attributes and cost factor components. That's why the proposed model includes fuzzy-sensitivity plots to analyse the effect of α as well as the degree of fuzziness, γ, in the candidate-alternative selection problem.

4. FORMULATION OF THE INTELLIGENT FUZZIFIED MCDM MODEL

4.1 Membership Function

There are 11 in-built membership functions in the MATLAB fuzzy toolbox. In the current study, a modified version of No. 7 MF has been used. All the built-in MF includes 0 and 1. In the current work, 0 and 1 have been excluded and the *S*-shaped membership function has been extensively modified accordingly.

As mentioned by Watada (1997), a trapezoidal MF will have some difficulties such as degeneration, i.e., some sort of deterioration of solution, while introducing fuzzy problems. In order to solve the issue of degeneration, we should employ a non linear logistic function such as a tangent hyperbolic that has asymptotes at 1 and 0.

In the current work, we employ the logistic function for the nonlinear membership function as given by

$$f(x) = \frac{B}{1 + Ce^{\gamma x}} \tag{11}$$

where B and C are scalar constants and γ, $0 < \gamma < \infty$ is a fuzzy parameter that measures the degree of vagueness, wherein $\gamma = 0$ indicates crisp. Fuzziness becomes highest when $\gamma \to \infty$.

Eq. (11) will be of the form as indicated by Figure 4 when $0 < \gamma < \infty$.

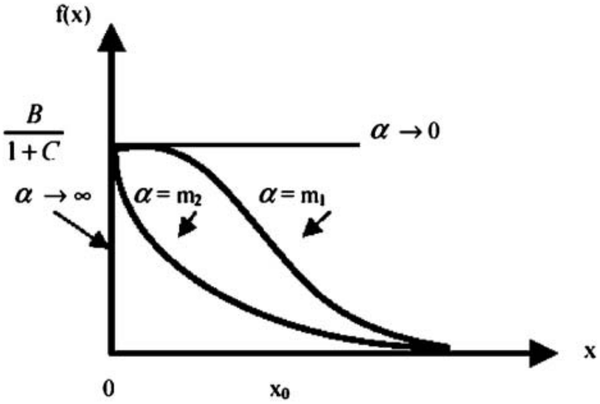

Figure 4. Variation of logistic MF with respect to fuzzy parameter, γ (where $m_2 > m_1$)

The reason why we use this function is that the logistic MF has a similar shape as that of the tangent hyperbolic function employed by Leberling (1981) but it is more flexible (Bells, 1999) than the tangent hyperbola. It is also known that a trapezoidal MF is an approximation to a logistic function. Therefore, the logistic function is very much considered an appropriate function to represent a vague goal level. This function is found to be very useful in making decisions and in implementation by the decision maker and implementer (Lootsma, 1997; Zimmerman, 1985; 1987).

Moreover, to avoid linearity in the real-life application problems, especially in industrial engineering problems, a non linear function such as modified MF can be used. This MF is used when the problems and its solutions are independent (Varela and Riberio, 2003). It should be emphasized that some nonlinear MFs such as *S*-curve MFs are much more desirable for real-life application problems than that of linear MFs.

The logistic function, Eq. (11), is a monotonically nonincreasing function, which will be employed as a fuzzy MF. This is very important because, due to an uncertain environment the availability of the variables are represented by the degree of fuzziness.

The said MF can be shown to be non increasing as

$$\frac{df}{dx} = \frac{BC\alpha e^{\gamma x}}{(1 + Ce^{\gamma x})^2} \tag{12}$$

An MF is flexible when it has vertical tangency, an inflexion point, and asymptotes. Since *B*, *C*, γ, and x are all greater than zero, $\frac{df}{dx} \leq 0$. Furthermore it can be shown that Eq. (11) has asymptotes at *f(x)* = 0 and *f(x)* = 1 at appropriate values of *B* and *C*. This implies:

$$-\lim_{x\to\infty}\frac{df}{dx} = 0 \text{ and } \lim_{x\to 0}\frac{df}{dx} = 0$$

These asymptotes can be proved as follows.

From Eq. (12), one gets

$$\lim_{x\to\infty}\frac{df}{dx} = -\frac{\infty}{\infty}.$$

Therefore, using L'hopital's rule, one obtains

$$\lim_{x\to\infty}\frac{df}{dx} = -\lim_{x\to\infty}\frac{B\gamma}{2(1+Ce^{\gamma x})} = 0 \tag{13}$$

As $x \to 0$, the situation is less vague and hence $\gamma \to 0$.
From Eq. (12), one gets

$$\lim_{x\to 0}\frac{df}{dx} = -\frac{BC\gamma}{(1+C)^2} = 0, \text{ when } \gamma \to 0 \tag{14}$$

In addition to the above equation, it can be shown that the logistic function Eq. (11) has a vertical tangent at $x = x_0$, x_0 is the point where $f(x_0) = 0.5$.

Furthermore it can also be shown that the said logistic function has a point of inflexion at $x = x_0$, such that $f''(x_0) = \infty$, with $f''(x)$ being the second derivative of *f(x)* with respect to *x*. An MF of *S*-curve nature, in contrast to linear function, exhibits the real-life problem.

The generalized logistic MF is defined as

$$f(x)=\begin{cases}1 & x<x_L\\ \dfrac{B}{1+Ce^{\gamma x}} & x_L<x<x_U\\ 0 & x>x_U\end{cases} \tag{15}$$

The *S*-curve MF is a particular case of the logistic function defined in Eq. (15). The said *S*-curve MF has specific values of *B*, *C* and γ. The logistic function as defined in Eq. (11) was indicated as an *S*-curve MF by Zadeh (1971; 1975).

4.2 Design of Modified, Flexible *S*-curve MF

To fit into the MCDM model in order to sense its degree of fuzziness, Eq. (15) is modified and redefined as follows and illustrated in Figure 5.

$$\mu(x)=\begin{cases}1 & x<x^a\\ 0.999 & x=x^a\\ \dfrac{B}{1+Ce^{\gamma x}} & x^a<x<x^b\\ 0.001 & x=x^b\\ 0 & x>x^b\end{cases} \tag{16}$$

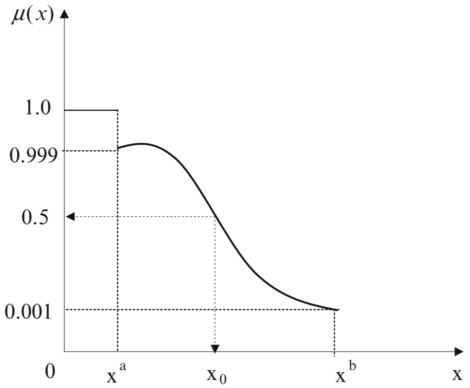

Figure 5. Modified *S*-curve membership function

We rescale the x-axis as $x^a = 0$ and $x^b = 1$ in order to find the values of B, C, and γ. Nowakowska (1977) has performed such a rescaling in his work on the social sciences.

The values of B, C, and γ are obtained from Eq. (16) as

$$B = 0.999\ (1 + C) \tag{17}$$

$$\frac{B}{1 + Ce^{\gamma}} = 0.001. \tag{18}$$

By substituting Eq. (17) into Eq. (18), one gets

$$\frac{0.999(1 + C)}{1 + Ce^{\gamma}} = 0.001. \tag{19}$$

Rearranging Eq. (19), one gets

$$\gamma = \ln \frac{1}{0.001}\left(\frac{0.998}{C} + 0.999\right). \tag{20}$$

Since B and γ depend on C, one requires one more condition to get the values for B, C, and γ.

Let, when $x_0 = \dfrac{x^a + x^b}{2}$, $\mu(x_0) = 0.5$.

Therefore,

$$\frac{B}{1 + Ce^{\frac{\gamma}{2}}} = 0.5, \tag{21}$$

and hence

$$\gamma = 2\ln\left(\frac{2B - 1}{C}\right). \tag{22}$$

Substituting Eq. (20) and Eq. (21) into Eq. (22), we obtain

$$2\ \ln\left(\frac{2(0.999)(1+C)-1}{C}\right) = \ln\ \frac{1}{0.001}\left(\frac{0.998}{C} + 0.999\right) \qquad (23)$$

$$\left(0.998+1.998C\right)^2 = C\left(998+999C\right) \text{ which in turn yields } \qquad (24)$$

Eq. (24) is solved and it is found that

$$C = \frac{-994.011992 \pm \sqrt{988059.8402 + 3964.127776}}{1990.015992} \qquad (25)$$

Since C has to be positive, Eq. (22) gives $C = 0.001001001$, and from Eqs. (17) and (22), one gets $B = 1$ and $\gamma = 13.81350956$.

Thus, it is evident from the preceding sections that the flexible, modified *S*-curve MF can be more easily handled than other nonlinear MFs such as the tangent hyperbola. The linear MF such as the trapezoidal MF is an approximation from a logistic MF and is based on many idealistic assumptions. These assumptions contradict the realistic real-world problems.

Therefore, the S-curve MF is considered to have more suitability in sensing the degree of fuzziness in the fuzzy-uncertain judgmental values of a decision maker. The modified *S*-curve MF changes its shape according to the fuzzy judgmental values of a decision maker and therefore, a decision maker finds it suitable to apply his/her strategy to MCDM problems using these judgmental values.

Thus the proposed *S*-shaped membership function is flexible due to its following characteristics:

(i) $\mu(x)$ is continuous and strictly monotonously nonincreasing;

(ii) $\mu(x)$ has lower and upper asymptotes at $\mu(x) = 0$ and $\mu(x) = 1$ as $x \to \infty$ and $x \to 0$, respectively;

(iii) $\mu(x)$ has inflection point at

$$x_0 = \frac{1}{\alpha}\ln\left(2+\frac{1}{C}\right) \text{ with } A = 1+C$$

The fuzzy intelligence of the proposed MCDM model is incorporated under a tripartite environment. A fuzzy rule-based decision (*if−then* rule) is incorporated in the algorithm to sense the fuzziness patterns under a disparate level-of-satisfaction of the decision maker. The aim is to produce a rule that works well on previously unseen data.

In the next chapter it will be demonstrated how to compute the degree of fuzziness and level-of-satisfaction, and a correlation among degree of fuzziness having a disparate level-of-satisfaction and the selection indices will also be elucidated to guide the decision maker in selecting the best candidate-alternative under an unstructured environment.

5. CONCLUSION

The proposed MCDM model shows how to measure a parameter called "level-of-satisfaction" of the decision maker while making any kind of decision. "Level-of-satisfaction" is a much-quoted terminology in classic as well as modern economics. To date, we are not aware of any reported research work in which level-of-satisfaction has been measured with a rigorous mathematical logic. The proposed model is a one-of-a-kind solution to incorporate the "level-of-satisfaction" of decision maker. Another solution can also be made with many sophisticated tools, like some approximation tool using neuro-fuzzy hybrid models.

The strength of the proposed MCDM methodology lies in combining both cardinal and ordinal information to get eclectic results from a complex, multi-person, and multi-period problem hierarchically. The methodology proposed in this chapter is very useful in quantifying the intangible factors in a good manner and in finding out the best among the alternatives depending on their cost factors. Contrary to the traditional way of selection using discounted cash flow (DCF), this methodology is a sound alternative to apply under an unstructured environment.

There may be some weaknesses due to the nonavailability of experts' comments, i.e., judgmental values. Comparison among various similar types of systems is the opportunity of the proposed model. An underlying threat is associated with the proposed model that a illogical decisions and mis-presentation of experts comments may lead to a wrong decision.

The MCDM methodology proposed in this chapter assumes that the decision is made under a fuzzy environment. A comparative study by accommodating different measures of uncertainty and risk in the MADM methodology may also be made to judge the best-suited measure of uncertainty. A knowledge-based system may be developed based on the modified AHP.

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